BMC2001

Abstracts of invited talks

Talks will take place in lecture rooms 1 and 2 of the Boyd Orr Building.

Monday 9 April 2001

6.00—7.00 pm

Plenary Speaker Clifford Taubes Lecture room 1

Title What we know and don't know about 4-dimensional spaces

A happy apex of ignorance has been reached in our understanding of smooth, 4-dimensional manifolds. I hope to explain how we reached this pleasant pinnacle and then shamelessly speculate on what the future (the millennium starting in 3000) might bring.

Tuesday 10 April 2001

9.30—10.10 am

Cameron Gordon Lecture room 2

Title Dehn surgery on knots

In 1910 Dehn showed that infinitely many homology 3-spheres with non-trivial fundamental group could be constructed by removing from the 3-sphere a solid torus neighbourhood of a knot K and sewing it back differently. This Dehn surgery construction of closed 3-manifolds from knots has been the focus of a good deal of attention over the last 25 years, and the general picture has emerged that, generically, the geometric and topological properties of the knot complement persist in the resulting closed 3-manifold. In particular, if the complement of K has a complete hyperbolic structure, then so do most of the manifolds obtained by Dehn surgery on K. There is some hope that it may be possible to completely classify the exceptions; we will describe current progress towards this goal.

Mark Jerrum Lecture room 1

Title Approximating the permanent (with Alistair Sinclair and Eric Vigoda)

The permanent of a matrix is a multivariate polynomial akin to the more familiar determinant, except that all monomials are given positive sign. In contrast to the determinant, which can be evaluated efficiently using Gaussian elimination, the permanent is known to be $\#P$-complete, even in the special case of a $0,1$-matrix. This classical result of Valiant almost certainly rules out a polynomial-time algorithm for computing the permanent of such a matrix *exactly*.

However, the question of whether the permanent of a $0,1$-matrix can be efficiently *approximated* has been open for some time. The question has recently been resolved positively. I shall present a "fully-polynomial randomised approximation scheme" for the permanent of a $0,1$-matrix (more generally, of an arbitrary matrix with non-negative entries). In the process, I hope to convey some of the techniques that are employed in the analysis of algorithms of this type, particularly in bounding the mixing time of complex Markov chains.

10.20—11.00 am

Nikolaos Bournaveas Lecture room 2

Title Low regularity solutions of the Klein-Gordon-Dirac equations

We discuss the relation between local existence of low regularity solutions of non-linear wave equations and global existence. To deal with low regularity solutions S. Klainerman and M. Machedon have developed in recent years powerful Harmonic Analysis techniques. We also discuss applications of these methods to the Klein-Gordon-Dirac equations.

Alexander Ivanov Lecture room 1

Title Local characterisations of graphs and geometries

Let Δ and Γ be graphs. Then Γ is said to be locally Δ if for every vertex $x \in \Gamma$ the subgraph in Γ induced by the vertices adjacent to x is isomorphic to Δ. The central problem to be discussed in the lecture is how to classify the graphs which are locally a given graph Δ. In general this problem is unsolvable in a certain explicit sense. On the other hand, if Δ belongs to a special class of graphs, like the class of strongly regular graphs, then the situation is much more optimistic. Of a special interest is the situation when Δ is the commuting graph of a conjugacy class of involutions in a group C. In this case the description of the graphs which are locally Δ is related to the classification problem of groups with the centralizer of an involution isomorphic to C. We will present a characterization of the Baby Monster sporadic simple group as the automorphism group of a graph which is locally the commuting graph of the central involutions in the Lie type group ${}^2E_6(2)$.

11.45 am—12.25 pm

Paul Martin Lecture room 2

Title A physicist's approach to algebraic representation theory

Recent interplay between the use of representation theory (typically, but not exclusively, algebraic Lie theory) in Statistical Mechanics, and the use in representation theory of ideas coming from the Physical context, is reviewed from a personal perspective (concentrating mainly on the latter direction).

Special Session on Partial Differential Equations

Jean-Yves Chemin Lecture room 1

Title Anisotropy in incompressible viscous fluids

The aim of this talk is to show how anisotropy can occur in incompressible viscous fluids and how the idea can be introduced in the problem of local (and global) well-posedness for incompressible isotropic viscous fluids. In particular, we shall prove local wellposedness and global wellposedness for small data in anisotropic spaces. The proof of those results uses in a crucial way the special form of the convective term.

2.00—2.40pm

Special Session on Partial Differential Equations

Pierre Collet   Lecture room 1

Title Epsilon entropy per unit volume and related quantities for dissipative PDEs in unbounded domains.

We will explain how the concept of epsilon entropy per unit volume can be applied to the attractor of some non linear partial differential equations of parabolic and hyperbolic type (with damping) in infinite domain. A natural extension leads to the definition of an epsilon topological entropy per unit volume. This last quantity initially defined in the continuum can be fully recovered using only data on a fine enough mesh. The hyperbolic case requires some special results in approximation theory.

2.45—3.25pm

Special Session on Partial Differential Equations

Emmanuel Grenier   Lecture room 1

Title Stability of boundary layers through Green's functions

The aim of the talk is to study the nonlinear stability of boundary layers of parabolic systems as the viscosity goes to 0, using a detailed analysis of the linearized system, and replacing energy methods by Green functions methods. This leads to sharp nonlinear stability results.

3.30—4.10pm

Special Session on Partial Differential Equations

Armen Shirikyan   Lecture room 1

Title Ergodicity for the randomly forced equations of mathematical physics

We discuss the large-time asymptotic behaviour of solutions for a class of dissipative PDE's perturbed by a random force. This class includes the 2D Navier-Stokes system with periodic or Dirichlet boundary conditions. Under the assumption that the noise is sufficiently non-degenerate, we show that the random dynamical system associated with the problem in question has a unique stationary measure. We also show that this measure is ergodic and obtain an estimate for the rate of convergence to it.

5.00—6.00 pm

Plenary Speaker Henri Berestycki   Lecture room 1

Title Propagation of fronts in periodic media

Reaction-diffusion equations arise in various contexts such as in combustion theory or in population dynamics in biology. This presentation is concerned with propagation phenomena in such excitable media. The classical works of Kolmogorov, Petrovsky and Piskunov, Aronson and Weinbeger, Fife and McLeod etc. deal with planar fronts. These are travelling fronts solutions in an homogeneous setting and are obtained as one dimensional solutions of some ordinary differential equations. After recalling some of these results, I will present some extensions of the notion of travelling front when there is an underlying periodic structure arising either from the geometry or from spatially periodic terms in the equation. I will discuss recent results about pulsating travelling fronts which are generalizations of planar fronts and lead to nonlinear partial differential equations.

Wednesday 11 April 2001

9.30—10.10 am

Kenneth Falconer   Lecture room 1

Title Mercerian theorems - old and new

Several classes of theorems concern relationships between limits of functions and their averages. Although Abelian and Tauberian theorems are widely studied, Mecerian theorems are less familiar. The lecture will present some Mercerian theorems, and relate them to dynamics and geometry.

Ian Leary   Lecture room 2

Title Doing things properly with metrics

I will define the universal proper G-space, $\underline{E}G$, for G a discrete group, and explain some of the reasons why one might be interested in it. One reason is its ubiquity: if you already know of a contractible space on which your favourite group G acts in an interesting way, then it is probably an $\underline{E}G$. I will show how ideas from metric geometry have helped in solving various questions concerning $\underline{E}G$ — questions which appear to lie within group theory or algebraic topology.

10.20—11.00 am

Ran Levi   Lecture room 1

Title Homotopy finite group theory

The main goal of this talk is to identify and study a certain class of topological spaces, which in many ways behave like p-completed classifying spaces of finite groups. We define a homotopy finite group to be a certain algebraic object - a small category which one can associate with a finite p-group, satisfying a number of axioms. The p-completed nerve of such objects are then shown to share many of the most important homotopy theoretic properties of p-completed classifying spaces of genuine finite groups. We also discuss some interesting examples of homotopy finite groups.

Robin Wilson   Lecture room 2

Title Victorian combinatorics

This talk will cover a range of combinatorial topics studied by British mathematicians in the second half of the 19th century – in particular, block designs (Kirkman, Anstice), partitions (De Morgan, Sylvester), graph theory (Cayley, Heawood), and enumeration (MacMahon).

11.45 am—12.25 pm

Oleg Kozlovski   Lecture room 2

Title Structural stability in one-dimensional dynamics.

In the talk we will discuss the Structural stability problem of one-dimensional dynamical systems. Though this problem is not solved yet even in 1D dynamics the simplest non trivial case is done: In the space of unimodal maps Axiom A maps are dense in the C^k topology.

Special Session on Modular Forms

Kevin Buzzard   Lecture room 1

Title   Slopes of modular forms

The eigenvalues of Hecke operators acting on classical modular forms are known to be algebraic integers. We will explain a conjecture, based on numerical evidence, that the p-adic valuations of these eigenvalues have some very precise structure. We also prove one special case of this conjecture.

2.00—2.50pm

Special Session on Modular Forms

Ernst-Ulrich Gekeler   Lecture room 1

Title   Observations about Eisenstein series for the modular group

Let E_k be the Eisenstein series of weight k ($k \geq 4$ even) for the modular group $SL(2, Z)$. Rankin and Swinnerton-Dyer have proved that their j-zeroes (i.e., the j-invariants of their zeroes as functions on the complex upper half-plane) lie in the real interval $[0, 1728]$ and possess a certain limit distribution. We present and discuss arithmetical properties (some shown, some only empirically observed) of these j-zeroes, which are also related with some apparently new congruences of the E_k.

3.00—3.50pm

Special Session on Modular Forms

Jacques Tilouine   Lecture room 1

Title   Siegel modular forms and Taylor-Wiles systems

In a joint work with Alain Genestier, we prove that, under several assumptions (among which minimal ramification), the p-adic representation associated to a rank 4 symplectic motive with Hodge weights $0, 1, 2, 3$ does come from a Siegel cusp form provided its reduction $\bmod p$ has large image and comes from a Siegel cusp form. The method is to construct Taylor-Wiles systems. The proof follows the lines of an unpublished work by Harris-Taylor, with several differences due to the non-properness of the Siegel varieties.

5.00—6.00 pm

Plenary Speaker   Henri Darmon   Lecture room 1

Title   Periods of modular forms and rational points on elliptic curves

If E is an elliptic curve over the rationals, the celebrated theorem of Wiles and Taylor completed recently by Breuil, Conrad, Diamond and Taylor asserts that E is attached to a modular form f on the Poincaré upper half plane. The most important Diophantine application of this result (n'en déplaise à Fermat...) arises from the theory of complex multiplication. This theory implies that the integrals of f between points belonging to a quadratic imaginary field K yield algebraic points on E with coordinates in certain class fields of K.

By replacing the line integrals of f by a notion of mixed period integral involving a blend of complex and p-adic integration, I have been led to conjecture an analytic construction of algebraic points on E, points which are defined over the class fields of real quadratic fields. The form of my conjecture raises the prospect of a generalisation of the theory of complex multiplication allowing the construction of rational points of elliptic curves from periods of modular forms, in a wide variety of situations which lie beyond the scope of the classical theory of complex multiplication.

Thursday 12 April 2001

9.30—10.10 am

Rob de Jeu Lecture room 2

Title Algebraic K–theory and special values of ζ and L–functions.

The classical relation in number theory between the residue at $s = 1$ of the ζ–function of a number field, and the regulator of the units of its ring of integers, was extended to give similar relations between the values of the ζ–function at all positive integers and higher regulators. We discuss this, and consider generalizations of this to curves. In the process, we also give a brief introduction to algebraic K–theory, and discuss how to describe it more explicitly.

Steffen König Lecture room 1

Title Schur algebras and representations of general linear and symmetric groups

Schur algebras are finite dimensional associative algebras describing the polynomial representation theory of general linear groups (over infinite fields of any characteristic). Structural properties of Schur algebras can be used to get explicit information on representations of general linear and also of symmetric groups, as the following examples indicate.

(1) Schur-Weyl duality relates representations of general linear and symmetric groups by a double centralizer property of Schur algebras and group algebras of symmetric groups. The abstract 'reason' for Schur-Weyl duality is the fact that Schur algebras have dominant dimension at least two.

(2) Decomposition matrices of general linear or symmetric groups have a kind of 'fractal' structure. A structural explanation for such repeating patterns is the fact that certain Schur algebras are isomorphic to subalgebras of other Schur algebras.

Various other algebras arising in algebraic Lie theory share structural properties with Schur algebras or group algebras of symmetric groups; among these are blocks of the Bernstein-Gelfand-Gelfand category of a semisimple complex Lie algebra, and Brauer algebras.

References

- Double centralizer properties, dominant dimension and tilting modules (joint with Inger Heidi Slungard and Changchang Xi), to appear in Journal of Algebra.

- Relating polynomial $GL(n)$-representations in different degrees (joint with Anne Henke), preprint.

- Enright's completions and injectively copresented modules (joint with Volodymyr Mazorchuk), preprint.

- The coinvariant algebra and representation types of blocks of category 0 (joint with Thomas Bruestle and Volodymyr Mazorchuk), to appear in Bull. LMS.

- When is a cellular algebra quasi-hereditary? (joint with Changchang Xi), *Math. Annalen* **315** (1999), 281-293.

- A characteristic free approach to Brauer algebras (joint with Changchang Xi), *Trans. AMS* **353** (2001), 1489-1505.

10.20—11.00 am

James McKee   Lecture room 1

Title   Snakes, stars, and Salem numbers

In a conference in 1997, Chris Smyth outlined a method of constructing Salem numbers and Pisot numbers from graphs. This talk will describe his construction, and some recent (2001) developments. One application of the construction is the result that for any integer t there exists a Pisot number with trace t. Achieving t negative is a challenge (it had been conjectured to be impossible): using "stars" alone, the smallest known degree for a Pisot number having negative trace is 23837; by introducing other graphs, this degree has been brought down to 1277.

Viacheslav Nikulin   Lecture room 2

Title   A theory of Lorentzian Kac–Moody algebras

We consider the problem of constructing a theory of Lorentzian (or hyperbolic) Kac–Moody algebras. This theory should be similar to the well-known theories of finite and affine Kac–Moody algebras.

As an example, we consider classification of some interesting Lorentzian Kac–Moody algebras of the rank three. Perhaps, this is the first example when a big class of Lorentzian Kac–Moody algebras was classified. This classification involves classification of hyperbolic root systems of the rank three which are appropriate for the theory of Lorentzian Kac–Moody algebras, and classification of meromorphic reflective automorphic forms with infinite product with respect to the paramodular group (the modular group for Abelian surfaces).

See details in math.AG/9810001, 0010329

11.45 am—12.45 pm

Plenary Speaker   Michel Broué   Lecture room 1

Title GL_n over a field with x elements (x an indeterminate)??

Many features of the group GL_n over a field with q elements, including its Sylow theory, character values, and modular representation theory, suggest that the group should be viewed as the specialization for $x = q$ of a mysterious object. The same applies to other reductive groups. Moreover, while the Weyl group of an actual reductive group must be a reflection group over Q, other reflection groups over the complex numbers apparently give rise to other mysterious objects. We shall present some evidence for these speculations.